Tonal and “Modal” Harmony:
A Transformational Perspective

Nicolas Meeùs, Keynote Address
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The approach to tonal harmony that forms the object of this communication is the Theory of Harmonic Vectors on which I have been working sporadically since more than 15 years. My intention today is to examine and discuss the relation between this theory and neo-Riemannian theory. One of the aims of my theory has been to evaluate the level of tonal affirmation in diatonic harmony; whether a less tonally affirmative harmony can be dubbed ‘modal’ (as is often done) will remain an open question.

My presentation will be divided in two parts. The first will be devoted to some theoretical considerations, comparing neo-Riemannian theory with Harmonic Vectors. The second part will propose a few examples of practical application and will confront the two theories and their results.

1. Theory

Let’s shortly review basic principles of neo-Riemannian theory. One aim of the theory is to describe movements of harmony in terms of parsimonious voice leading. Any triadic progression can be described as resulting from a more or less complex combination of elementary melodic movements. It is inherent in the structure of triads as piles of thirds that these elementary movements are conjunct: any note of the chromatic scale that does not belong to a given triad is adjacent to at least one of its notes. The melodic movements may take one of three forms of conjunct second, the tone, the diatonic semitone and the chromatic semitone, and each of these can lead from one triad to another:
• the 5^{th} of a major triad can move up a tone to become the prime of a minor triad, or inversely: it is the R relation, linking a triad to its relative;
• the third of a major triad can move down (or up) a chromatic semitone: it is the P relation, linking two triads on the same root but of opposed mode — i.e. leading a triad to its “parallel” one;
• and the prime of a major triad can move down a diatonic semitone to form a minor triad, or inversely: it is L relation, the “leading-tone” relation.
Neo-Riemannian relations are reversible: they are in operation in either direction.

Harmonic Vectors envisage harmonic progressions in terms of root (or fundamental bass) progressions. To each of the three elementary neo-Riemannian relations corresponds a characteristic movement of the fundamental bass – which, merely duplicating one of the notes of each triad, can be considered a “ghost” bass, a mere symbolic representation of the movement between the triads:
• down (or up) a minor third for the R relation;
• no movement of the fundamental bass for the P relation;
• up (or down) a major third for the L relation.

There is a tendency, in some of our circles, to oppose theories of voice leading to those based on root progressions. This, to me, is a misconception. I believe on the contrary that the two types of theories are closely linked: a description of a fundamental bass movement is nothing else than a synthetic description of the voice leading above it. The neo-Riemannian R relation essentially is a description of a root movement from a given triad to its relative (major or minor). *Leittonwechsel*, the “change to or from the leading note”, the neo-Riemannian L relation, apparently refers more specifically to a movement in the voice leading, but Riemann himself certainly understood it as a root movement. In his terminology, the word *Wechsel* refers to the change of mode of the triad, from minor to major or from major to minor. For him, both the R and the L transformations are *Terzwechsel*, “changes of a third”, i.e. root movements.[1]

Let’s sum up:

• the R relation is a minor third relation, descending from major to minor or ascending from minor to major. The terminology itself stresses both the fundamental bass relation and its reversibility: the relation, in either direction, is from a triad to its relative.

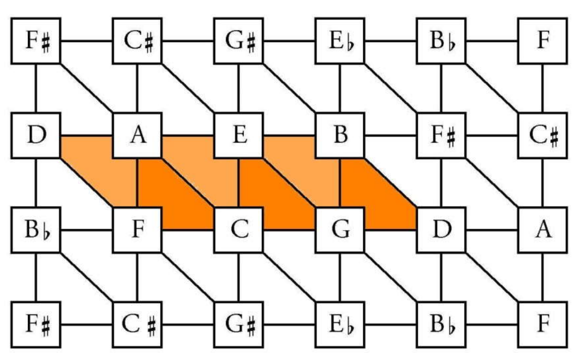
• the P relation involves no movement of the fundamental bass and leads in either direction to the parallel triad;

• the L relation is a major third relation, ascending from major to minor or descending from minor to major.

It may be noted that the P operation is the only one that involves chromaticism; the two others link triads that belong to one and the same diatonic scale: they are *Leitereigen*; they form, of course, the basic relations within a diatonic harmony.

* * *

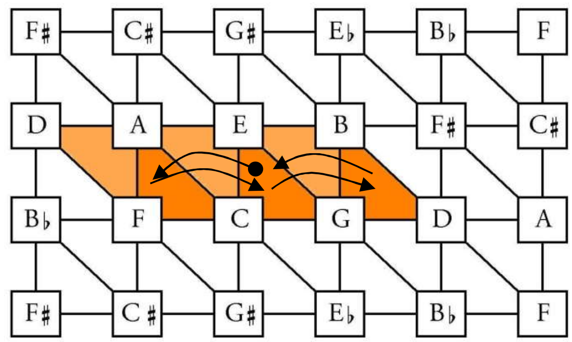
The diatonic scale, the *diatony*, can be described in the *Tonnetz* as formed by three major triads distant from each other by a fifth. More specifically, this is a representation of the diatonic system in just intonation. In the *Tonnetz*, just major thirds are represented on the vertical axis and just fifths on the horizontal axis, and Leonard Euler imagined it to illustrate just intonation. The network may be completed with the addition of three minor triads, minor relatives to the three major ones. This, however, results in one of the degrees being present twice (*D*, in the case represented hereby). In just intonation, *D* in the *d* minor chord (at the left of the figure) supposedly is a comma lower than that in the *G* major chord (at the right), a fact that led to the curious statement, by Simon Sechter and by Anton Bruckner (the latter derided by Schenker) that the chord of the II^{d} degree (*d* minor, here, in *C* major) had to be



[1] Riemann’s terminology cannot be further discussed here, because his dualistic view of harmony led him to deduce the fundamental bass of minor chords in an usual way.

considered a dissonance and treated as such. Such considerations should not concern us here, as we today conceive the *Tonnetz* in terms of equal temperament; we will nevertheless have to come back on this point.

It is immediately obvious that the possible movements of the harmony or the voice leading within the restricted area of the diatony remain extremely limited. They consist in alternating R and L transformations, leading, from left to right, by an R transformation (a major tone movement from *D* to *C*), transforming the *d* minor triad into its relative the *F* major one, then by an L transformation (diatonic semitone from *F* to *E*) transforming *F* major into *a* minor, then an R relation (*A* to *G*) to C major, L (*C* to *B*) to e minor, and R *E* to *D*) to *G* major. The circulation of course can be reversed, leading from *G* major at the right to *d* minor at the left by a similar succession of R and L transformations, i.e. by the same parsimonious movements in the reverse direction. Riemann may have thought that tonal harmony worked in that way, when he described the "full cadence" in major as going from Tonic to Subdominant to Tonic to Dominant to Tonic (T–S–T–D–T), a movement that could be represented in the *Tonnetz* as in the figure hereby. These movements are root movements of a fourth or a fifth: in other words, they represent RL or LR neo-Riemannian transformations. This is not, however, how tonal harmony usually works.



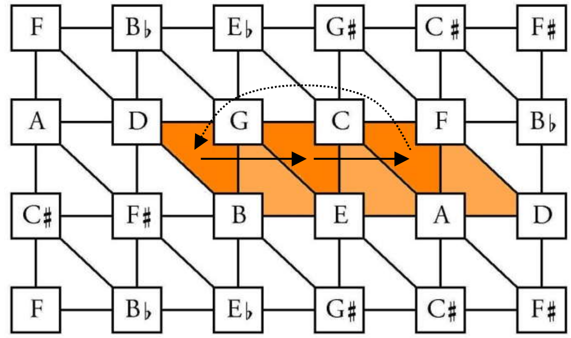
* * *

Before going further, I will have to modify the image of the *Tonnetz*: I will mirror it from left to right, because, as we will see, tonal progressions usually develop in one single direction, and reading the *Tonnetz* will be more comfortable if that direction is the usual reading direction, from left to right. I will also flip it from top to bottom, so as to represent the loss of a comma in the descending direction from each horizontal line to the next (in Euler's presentation of the *Tonnetz*, on the contrary, each horizontal line conceptually is a comma lower that the line below it.

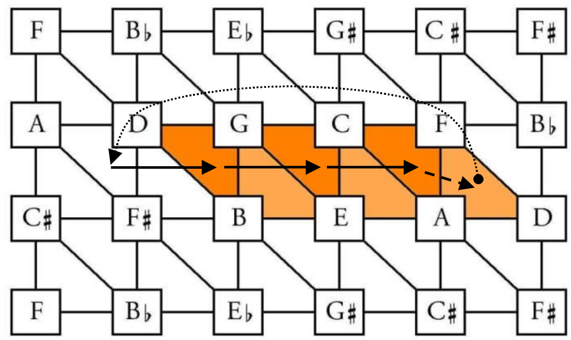
* * *

Some of the most common tonal progressions in the major mode are down a fifth from major to major, as from V to I of from I to IV. They obey what Schenker calls the *Quintengeist der Stufen*, the tendency of the degrees to move by (descending) fifths. It may however often be difficult to determine whether the progression is V–I or I–IV, unless a third chord is present. In other words, the progression may have a quality which remains independent of the chords concerned: this is what I call the "dominant vector", the function of the descending fifth or ascending fourth progression. The progression in the reverse direction, IV–I or I–V, is the "subdominant vector". What makes this view transformational is that it deduces the functional labels from the transformations or from the root movements, instead of deducing the transformations or the root movements from the functions and from the attractions that they supposedly create; the functions are not considered to reside in individual chords, but in a relation between chords. The *Quintengeist der Stufen* does not result from the function of the *Stufen*; on the contrary, it is the origin of these functions.

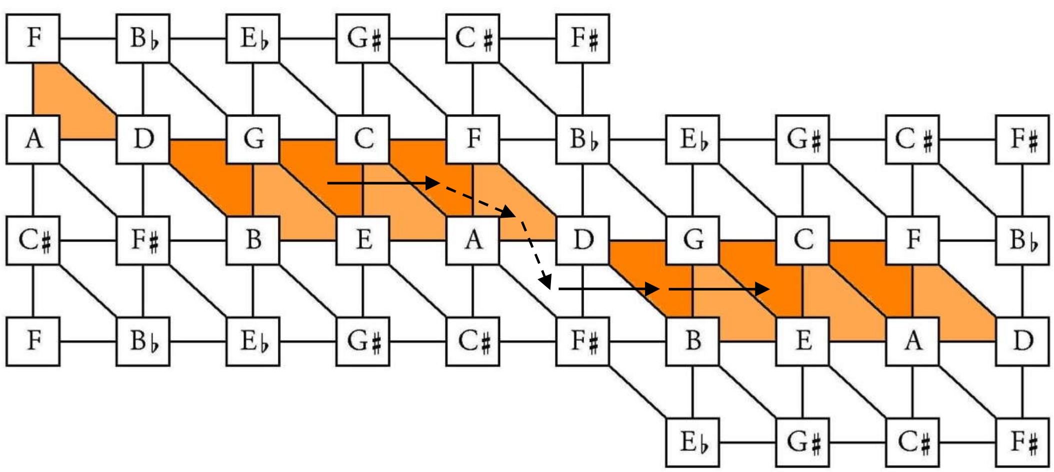
Tonality is a circulation, a circular movement, and one fundamental question about tonal progressions is how one comes back to the tonic – i.e., how one turns back from subdominant (a dominant vector *after* the tonic) to the dominant (a dominant vector *before* the tonic). This perplexing question has formed a constant preoccupation of all theories of tonality. August Halm, who deserves our consideration for having been in friendly terms with both Riemann and Schenker, stressed the *abyss* between the Subdominant and Dominant degrees.



One most interesting solution of this problem is that of Rameau's *double emploi*, which first notes the kinship between the subdominant chord (*F*) and its minor relative (*d*) (a neo-Riemannian R relation), then the relation from the subdominant's minor relative (*d*) to an implicit dominant's dominant (*D*) (a P relation), leading by a new descending fifth to the dominant itself (see the figure hereby). All subsequent theoretical description of the subdominant–dominant progression may be considered variants of Rameau's, usually involving some kind of implicit P transformation which necessarily exceeds the limits of strict diatony.

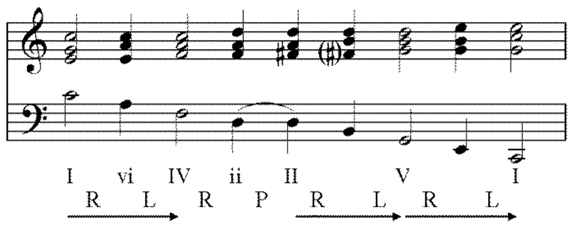


What remains unclear in this graphic description, however, is the P transformation from the subdominant's relative (*d* minor) to the dominant's dominant (*D* major), for which the figure does not propose a simple path. The transformation from Subdominant to Dominant should better be illustrated as in the figure hereunder. This invites reminding that the *Tonnetz* is toroidal and that its configurations repeat in a circular way, so that I will once again have to redraw it as in the following figure, where the diatonic area appears twice – the one at the right being a comma lower than the one at the left, in just intonation at least.



What this figure shows is a tonal "functional cycle", a T–S–D–T succession. The first succession, from T to S, is a simple RL relation, a dominant vector, implicitly passing through the tonic's relative, *C* (R) *a* (L) *F*. The second is more complex, as already mentioned, leading by a P transformation from the subdominant's relative to the dominant's dominant, then, by a new RL relation (a second dominant vector), to the dominant, *F* (R) *d* (P) *D* (R) *b* (L) *G*. After this, the return to the tonic is once again a mere RL relation (a third dominant vector), passing through the dominant's relative (e minor). In just intonation, the final tonic would be a comma lower than the initial one.

The "functional cycle" described here produces what may be termed the *tonal omnibus*, a succession of triads from tonic to tonic using the most straightforward neo-Riemannian relations, i.e. a parsimonious voice leading, *C* (R) *a* (L) *F* (R) *d* (P) *D* (R) *b* (L) *G* (R) *e* (L) *C* (see below). The central P relation is striking, which corresponds to Rameau's *double emploi*; it is this relation that makes the tonal return possible.



The combined RL relations form dominant vectors, as indicated by the arrows. I have chosen in this representation to write the four main chords, I, IV, V and I, as half notes; but they might be substituted by others of the series, producing successions such as I – ii – V – I or I – vi – ii – V – I, etc. Two triads have been given no roman numeral, because they are not normally used in tonal progressions of this type; they correspond to degrees vii and iii. The *tonal omnibus* consists in a descending series of thirds: *C a F d D b G e C* (with a double *d*/*D* in the middle). Several points must be noted:

• One relation on two is an R relation, producing an almost continuous RL alternation.

• There is a regular alternation of major and minor triads (as is normal in a succession of Riemannian *Wechsel*), but the descending chain of thirds is interrupted by the P relation, the double *d* / *D*, which ensures the possibility to return to the starting point.

• Both the R and the L relations form descending thirds: the R relation a minor third from major to minor, the L relation a major third from minor to major.

• All the parsimonious voice leading movements are ascending, a point strikingly contradicting Schenker's choice of a descending line as the fundamental line of the tonal affirmation. This point, which won't be further discussed here, indicates that the voice leading suggested by the neo-Riemannian relations is but an abstract one, not necessarily realized as such in the music itself.

One might imagine an abstract model of the *omnibus*, consisting in a regular alternation of descending R and L relations, forming an infinite succession which inexorably leads flatwards, never returning to the tonal center.

… *E c♯ A f♯ D b G e C a F d B♭ g E♭ c A♭* …

The tonal circulation is made possible by a return backwards (sharpwards), consisting in a P relation (here from *d* to *D*), Rameau's *double emploi*. Positioning the P relation on the second degree of the scale results in the least trespassing of the limits of the diatony, with only one implicit chromatic degree. It is on this abstract model that the few examples to be discussed now will be based.

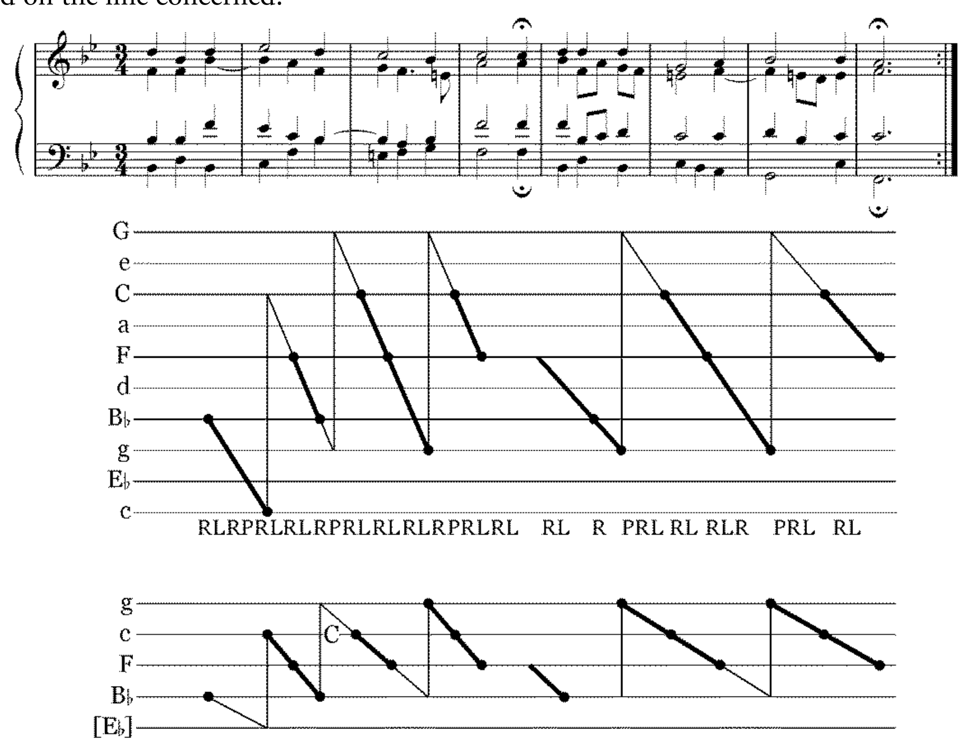
2. Practical application

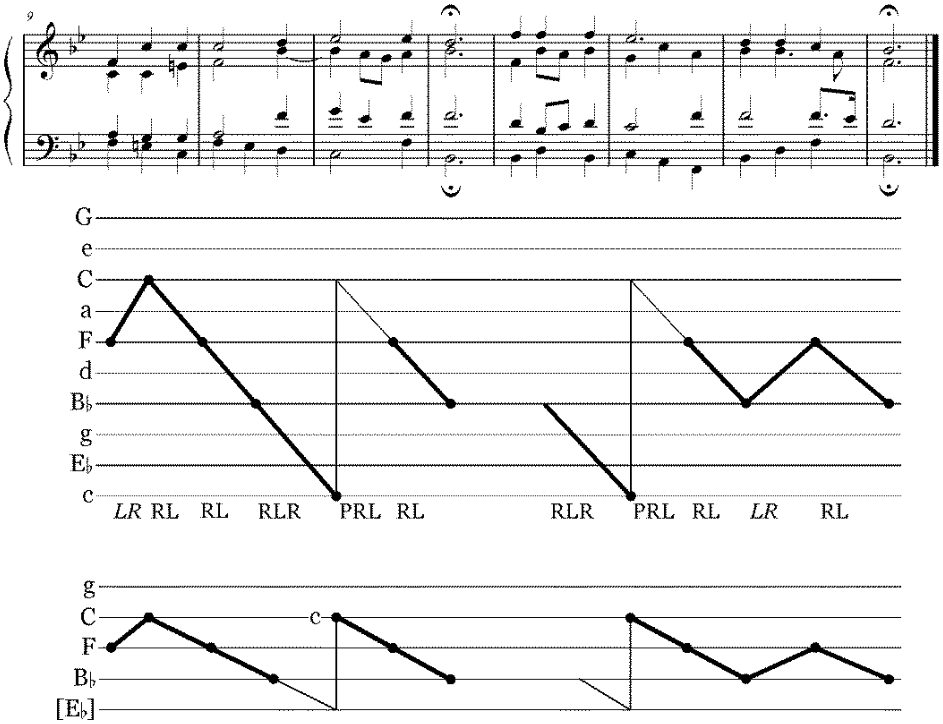
The pieces proposed for the analysis are Bach chorales: they may not form the best choice for illustrating how tonality works, but they fit the short time available in that they display many chords and chord progressions in few bars.

2.1. J. S. Bach, Choral *Gottlob, es geht nunmehr zu Ende*, BWV 321

The first set of lines under the score describes the abstract model of the *omnibus* through which the fundamental bass meanders. On this set of lines, the fundamentals can be inscribed as points, taking account of the mode of the triads; this is equivalent to a ciphering in Roman numerals, however with the important difference that here it is not necessary to determine the tonality beforehand. It then remains to connect the points in order to determine the neo-Riemannian relations between them. As we have seen in the *Tonnetz*, the diatonic area covers six triads, three major ones and their three relative minor ones: a harmonic succession remains diatonic as long as it does not exceed six lines of the graph, from a thick (major) one above (sharp side) to a thin (minor) one under (flat size). In *B♭* major, the diatony would extend from *F* to *c*. The successive neo-Riemannian relations are inscribed under the set of lines, in roman letters for the movements corresponding to those described in the *omnibus* above, in italics for those in the reverse direction.

The second set of lines under the score presents a more compact representation of the root movements (and of the corresponding theoretical parsimonious voice leading), based on the cycle of fifths: this corresponds to Harmonic Vectors. In order to obtain this representation, it has been necessary to neglect the distinction between major and minor triads, i.e. to reckon the roots without considering the mode of the triads above them. In this example, only the *C* chord exists both in its major and minor forms, as indicated by the change from *c* to *C*, then from *C* to *c*, inscribed on the line concerned.





J. S. Bach, *Choral Gottlob, es geht nunmehr zu Ende*, BWV 321,
and analysis

The first neo-Riemannian relation, from *B♭* major to *c* minor in bars 1-2, is a RLR relation. The most parsimonious path for the next relation, from *c* minor to *F* major, implies a P relation from *c* minor to *C* major, followed by an RL relation to *F* major. Because such shifts usually happen on the II^{d} degree, the P relation is a strong indication to the fact that the key is *B♭* major. The next relation is a simple descending fifth, from *F* to *B♭*, an RL relation. The following root movement, from *B♭* major to *C* major, is again raising sharpwards and involves a more complex path, L from *B♭* major to *g* minor, then P from *g* to *G*, then RL from *G* to *C*. The P relation from *g* to *G* seems to indicate *g* as the II^{d} degree and suggests a shift to the key of *F* major – this is but another way to acknowledge *e♮* in the *C* major chord. The following progressions are RL from *C* to *F*, then RLR from *F* to *g*. A new *double emploi* follows, from *g* to *C*, PRL, with the P relation again on *g*, confirming the modulation to *F* major. The second phrase of the choral can be described in the same way; it includes the same root progressions and confirms the key of *F* major. These first two phrases conform exactly to the theoretical description given above: they circulate the chain of thirds flatwards, with occasional commatic shifts (P relations) that regain a higher position sharpwards. One relation on two is an R, always from major to minor, and the even relations between them are either L or P, always from minor to major. No usage is made of the neo-Riemannian relations in the reverse direction.

The description on the second set of lines, corresponding to the cycle of fifths, also makes use of a traditional idea of the theory of harmony, that of *substitution*, the idea that a given chord can stand for another. In the first root movement, from *B♭* to *c*, for instance, *c* is taken to substitute for its major relative, *E♭*, as the result of an implicit descending fifth. This is another way of describing Rameau's *double emploi*, considering for instance that in the succession I-ii-V-I, the chord of ii, is a substitution for IV with respect to the preceding I; or else, in the succession I-IV-V-I, the chord of IV is a substitution for ii. This also is the essence of the Riemannian *Terzwechsel* which forms the basis of neo-Riemannian theory. For Riemann, a chord of the ii^{d} degree is a subdominant because it forms a *Parallelwechsel*, an R relation, with the chord of IV, etc.

The following movement, *c* to *F*, can be expressed as a mere falling fifth, neglecting the fact that it leads from a minor to a major chord. *F* to *B*♭ is a straightforward falling fifth from major to major. From *B*♭ to *C*, the root movement must be explained by an implicit *g* minor chord, substituted for *B*♭, leading to *C* by a falling fifth from minor to major. The capital *C* on the *C* line indicates that the triad on this root now is major. Etc. The figure so obtained is but a compact version of the one above it. It is in this sense that my own theory closely resembles a neo-Riemannian theory.

The second part of the choral shows basically the same disposition: the neo-Riemannian P relations are again from *c* to an implicit *C*, corresponding to the return to *c* minor in the presentation according to the cycle of fifths and indicating a return to the key of *B*♭ major; there are two ascending fifths, in m. 9 and 15, corresponding to *LR* relations; these are in the reverse order from the more normal RL relations, and indicated by italic letters.

All this invites statistics on the usage of the different neo-Riemannian transformations and/or the various root movements, as in the following tables which present results first from a neo-Riemannian point of view, i.e. based on chains of thirds, second from the point of view of Harmonic Vectors, based on the cycle of fifths. In the statistics of neo-Riemannian relations, however, it has been necessary to make a distinction between the relations in the "normal" direction (R from major to minor, P and L from minor to major) and those in the reverse direction: neo-Riemannian theory considers the relations as reversible, but that is not true in the case of tonal harmony.

The tables at the left evidence the highly asymmetric distribution of the neo-Riemannian relations and of the Harmonic Vectors, in each case above 90% for the directions described as "normal" in tonal harmony. The tables at the right consider pairs of relations or of root movements. The asymmetry is particularly clear in the lower table, where it appears that all pairs of root movement include one progression +4; all SV (-4) are preceded and followed by +4, which means that they are all involved in a "pendular" movement of the fundamental bass.

Neo-Riemannian transformations

R	47 %	94%
L	36 %	
P	11 %	
R	3 %	6 %
L	3 %	
P	—	

	R	L	P	*R*	*L*	*P*
R		37%	11%			
L	32%				3%	
P	11%					
R	3%					
L				3%		
P						

Harmonic vectors

$+4^{th}$	65 %	Dominant: 92 %
$+2^{d}$	23 %	
-3^{d}	4 %	
-4^{th}	8 %	Subdominant: 8 %
-2^{d}		
$+3^{d}$		

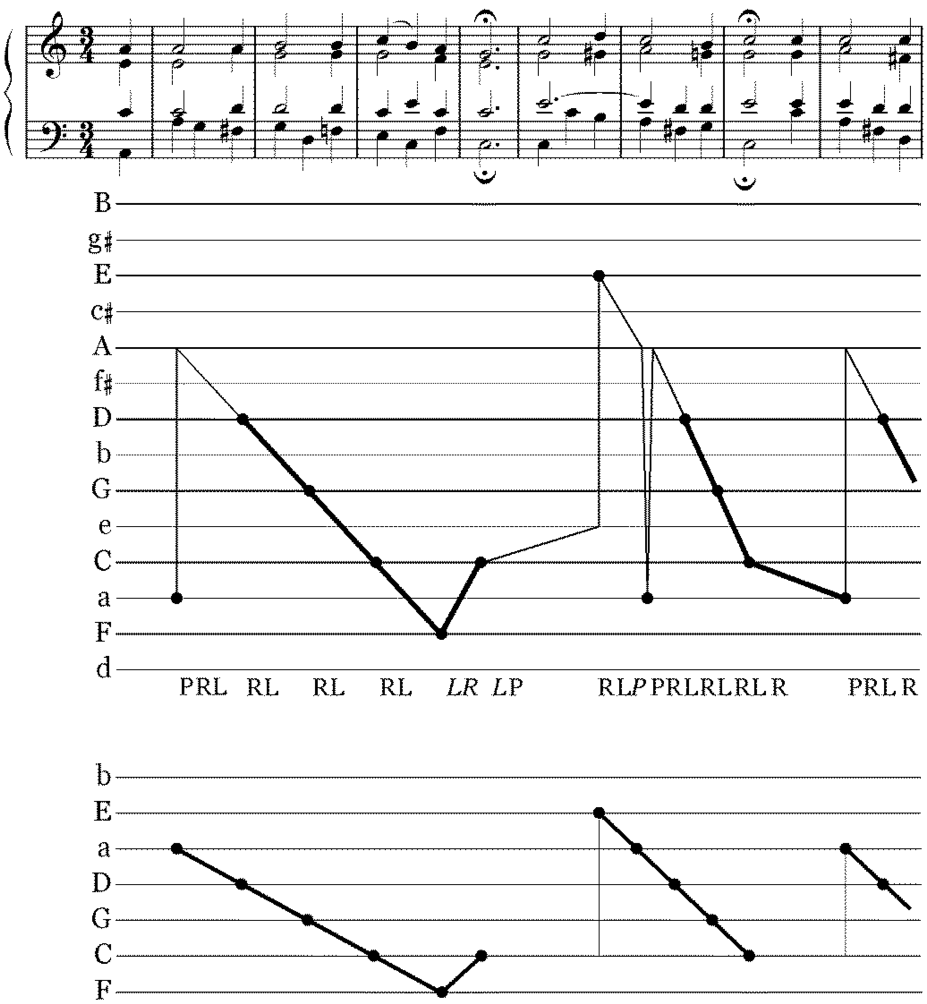
	$+4^{th}$	$+2^{d}$	-3^{d}	-4^{th}	-2^{d}	$+3^{d}$
$+4^{th}$	32 %	20 %	4 %	8 %		
$+2^{d}$	24 %					
-3^{d}	4 %					
-4^{th}	8 %					
-2^{d}						
$+3^{d}$						

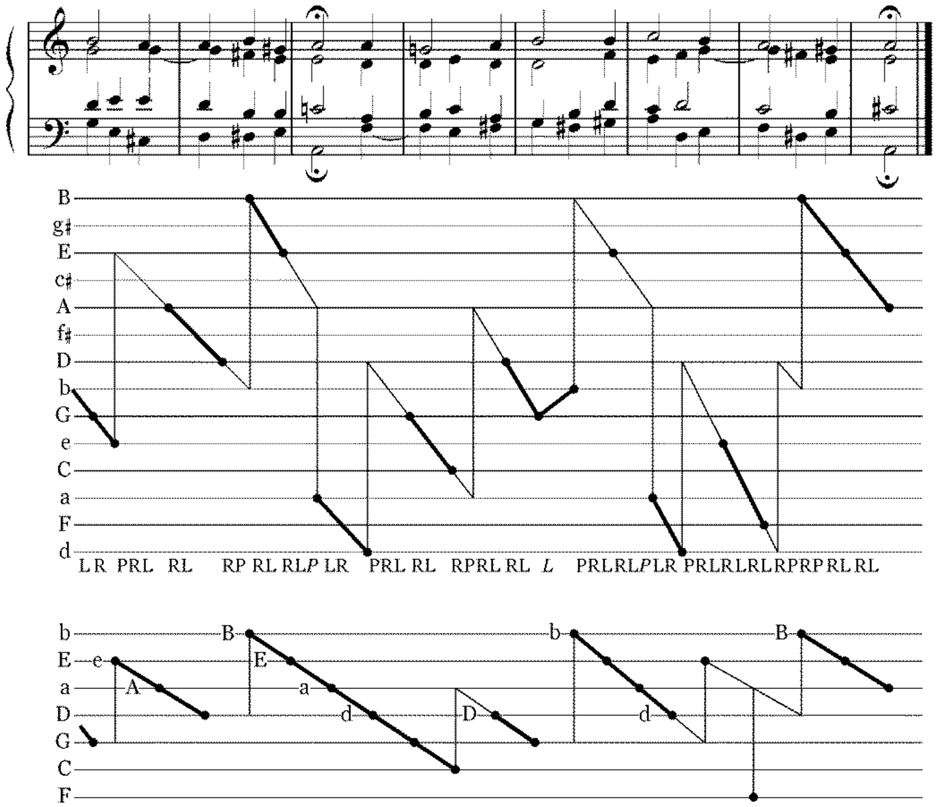
Statistics for BWV 321

2.2. J. S. Bach, Choral *Puer natus in Bethleem*, Breitkopf 12

This is an example of a choral in minor. The presentation is the same as above. One will note:
— the wider range covered both in the cycle of thirds and the cycle of fifths, expressing a rapid change from the minor key to its major relative.
— triads of the "wrong" mode in the cycle of thirds, such as the a (tonic) triad in mes. 6. The presentation according to the cycle of fifths smooths that out.

In the second part, a comparison between the cycle of thirds and the cycle of fifths evidences that successive roots may be in the "wrong" mode, e.g. *B-E* in mes. 10 (for *b-e*), or *a-d* in mes. 11 (for *A-D*). Again, that is smoothed in the second presentation (cycle of fifths). The statistic results remain surprisingly similar to those in major, with only a small increase in the number of different successions of transformations or vectors.





J. S. Bach, Choral *Puer natus in Bethleem*, Breitkopf 12,
and analysis

Neo-Riemannian transformations

R	42 %	91 %
L	34 %	
P	15 %	
R	1 %	9 %
L	4 %	
P	4 %	

	R	L	P	*R*	*L*
R		32%	10%		
L	27%				3
P	15%				
R					1
L			3%	1%	
P		3%	1%		

$+4^{th}$	72 %	Dominant: 91 %
$+2^{d}$	6 %	
-3^{d}	13 %	
-4^{th}	3 %	Subdominant: 9 %
-2^{d}		

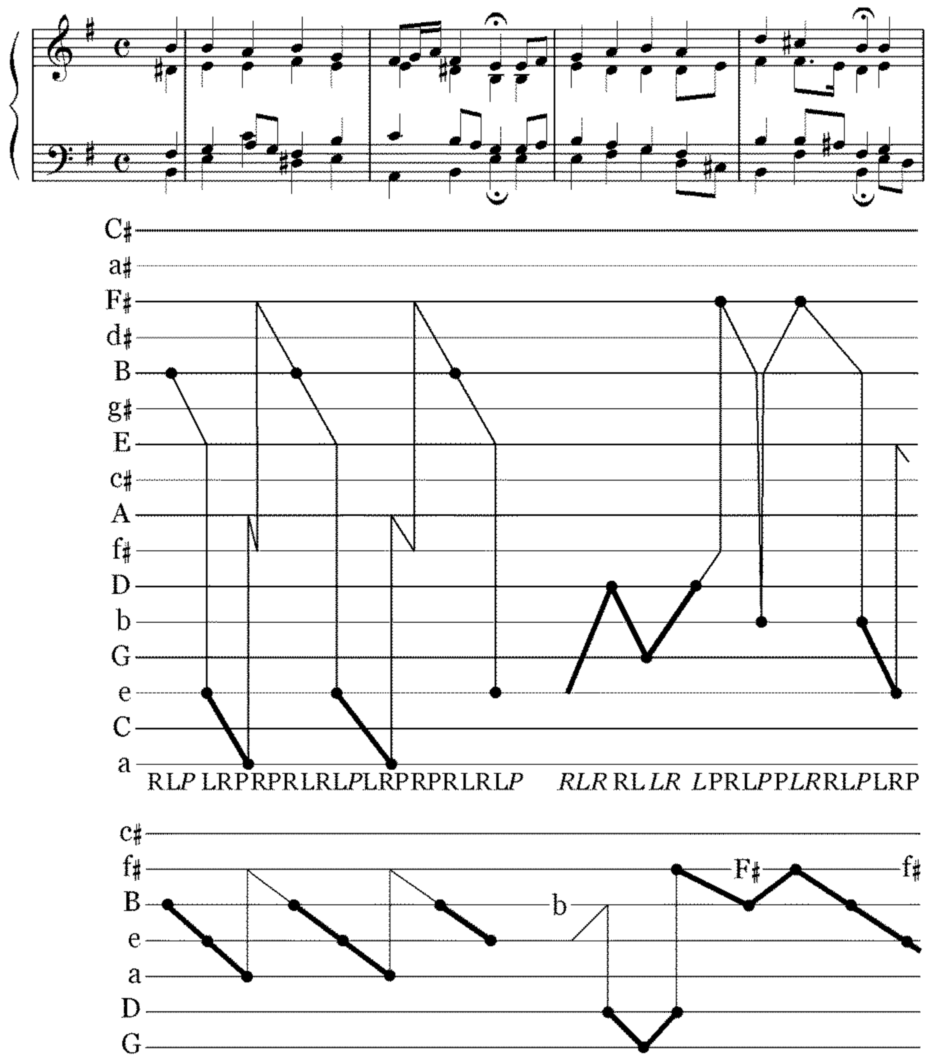
	$+4^{th}$	$+2^{d}$	-3^{d}
$+4^{th}$	50%	3%	13%
$+2^{d}$		3%	
-3^{d}	13%		
-4^{th}			
-2^{d}			

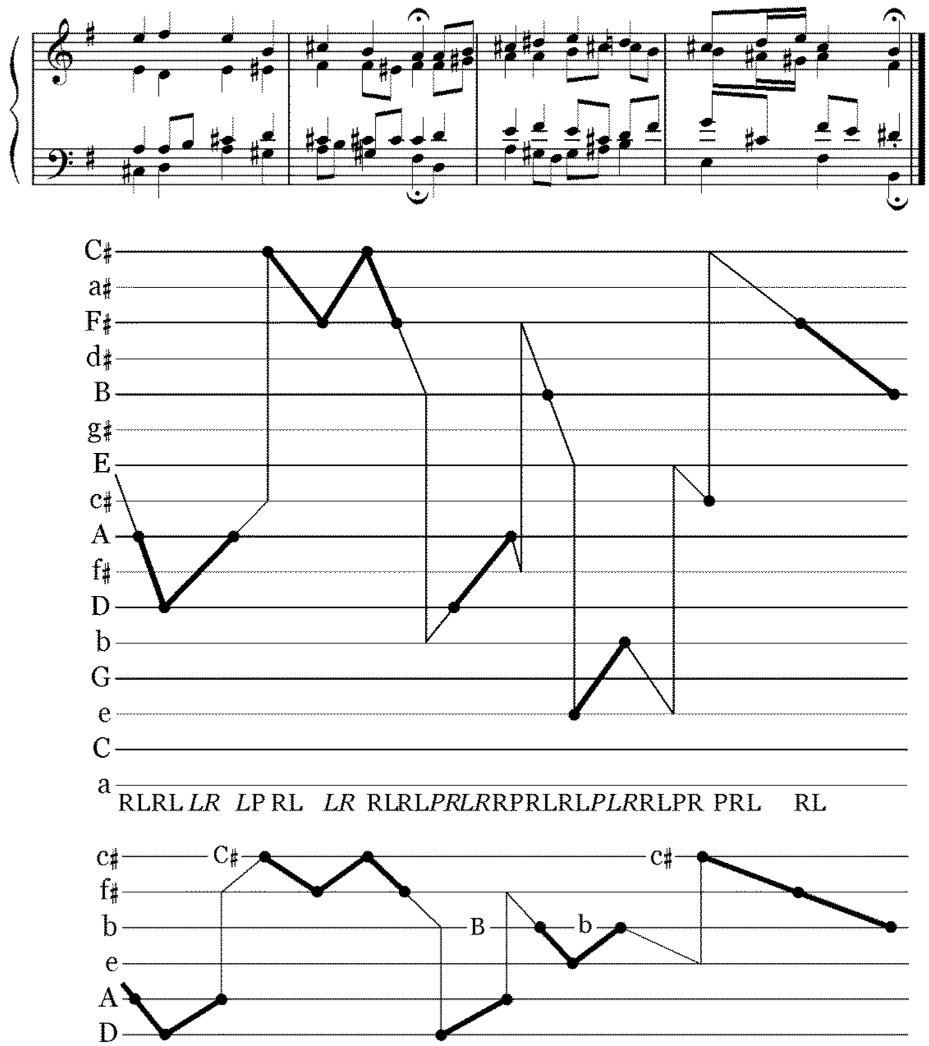
Statistics for *Puer natus in Bethleem*

2.3. J. S. Bach, Choral *Herr Jesu Christ, mein's Lebens Licht*, BWV 335

This choral is of a type sometimes qualified "modal". This may not be a very fitting description, but I won't discuss this aspect now. What the analyses evidence, in any case, is that the logic of the harmony is different from that of the previous examples and, as will appear later, shows some analogy with that of earlier pieces for which the "modal" qualification might be considered more satisfying.

The first phrase, mes. 1-2, despite its somewhat surprising appearance in the cycle of thirds, remains quite normal, as shown in the cycle of fifth. The major chord of *B*, dominant in *E* minor, provokes a situation which exceeds the limits of the diatony. In the following phrases, similarly, several chords and several progressions appear shifted with respect to the diatony, either because of a change of mode (P relation) in the cycle of thirds, or by a substitution in the cycle of fifths.





J. S. Bach, Choral *Herr Jesu Christ, mein's Lebens Licht*, BWV 335,
and analysis

The statistics are drastically different from the preceding ones, with figures of the order of 70 % of neo-Riemannian relations in the "normal" direction or of dominant vectors, against 30 % in the reverse direction (instead of 90 % against 10 % in the examples above). One notes however that the pairs of "normal" neo-Riemannian relations (57 %) or of dominant vectors (48 %) remain the most frequent, even if much less so than in the preceding examples.

Neo-Riemannian transformations

R	30 %	
L	26 %	69 %
P	13 %	
R	11 %	
L	11 %	31 %
P	9 %	

	R	L	P	*R*	*L*	*P*
R		23%	9%			
L	12%				3%	9%
P	13%				1%	
R	6%				5%	
L			3%	9%		
P		4%	1%	3%		

Harmonic vectors

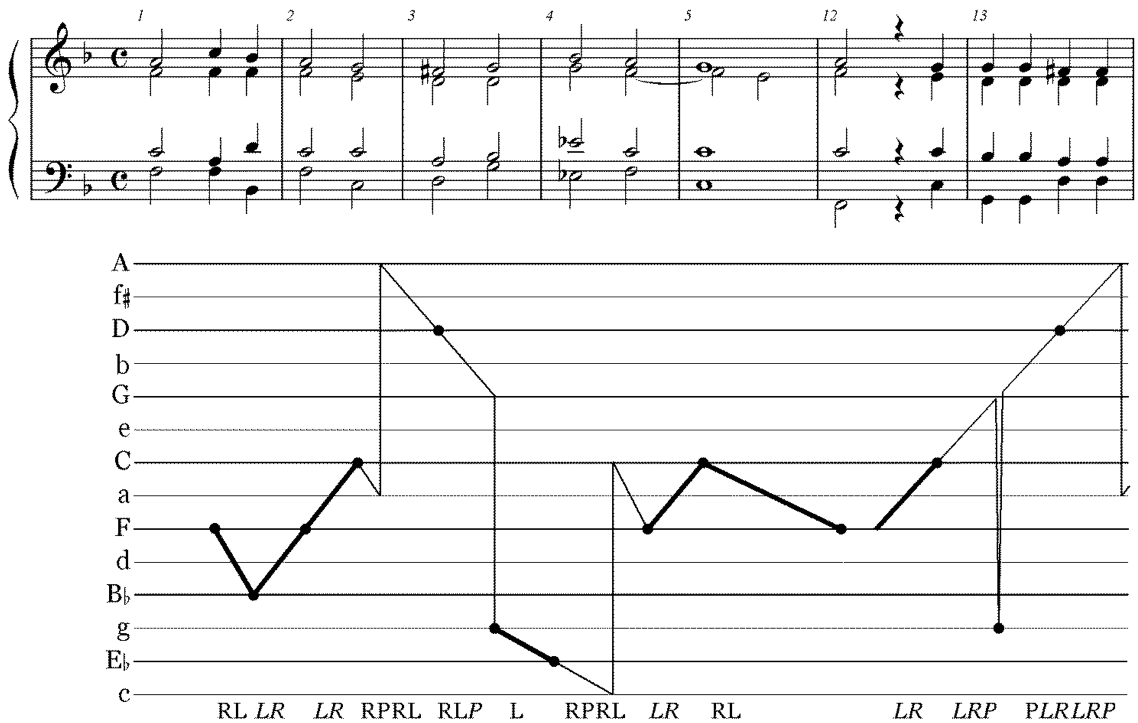
$+4^{th}$	53 %	Dominant:
$+2^{d}$	13 %	69 %
-3^{d}	3 %	
-4^{th}	20 %	Subdominant:
-2^{d}	3 %	30 %
$+3^{d}$	7 %	

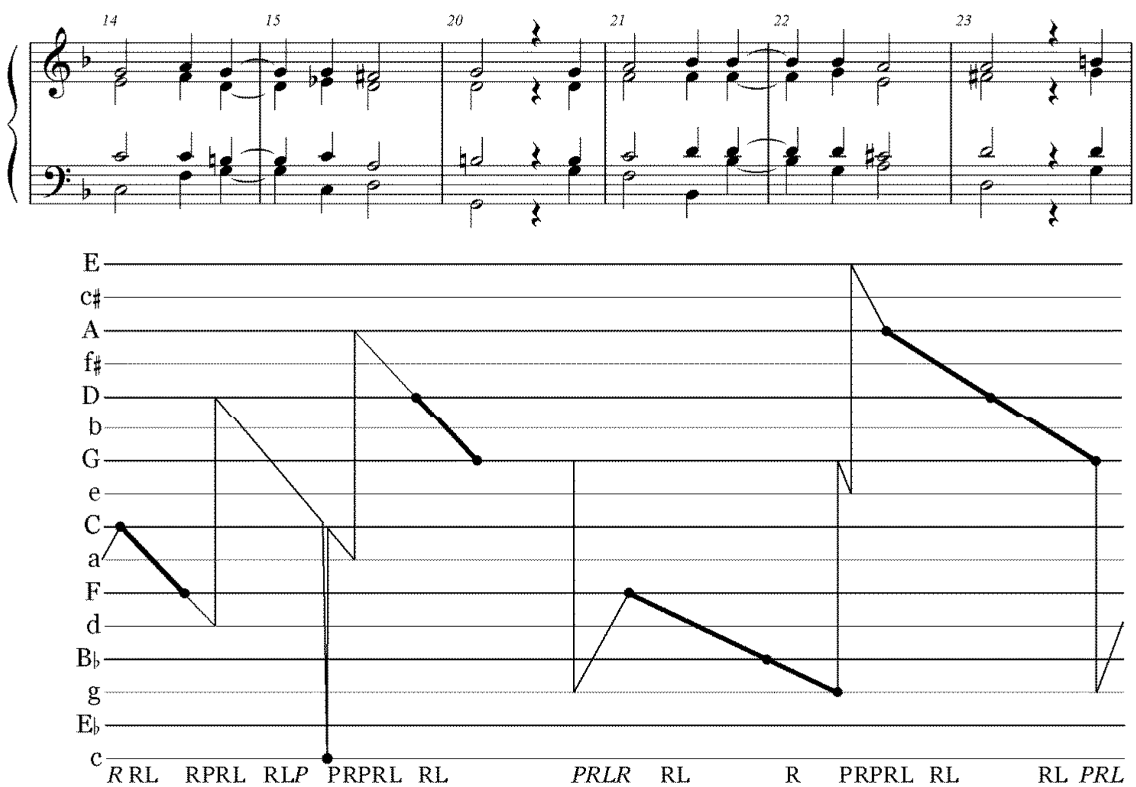
	$+4^{th}$	$+2^{d}$	-3^{d}	-4^{th}	-2^{d}	$+3^{d}$
$+4^{th}$	22%	7%	4%	19%	4%	
$+2^{d}$	15%					
-3^{d}				4%		
-4^{th}	7%					7%
-2^{d}	4%					
$+3^{d}$	7%					

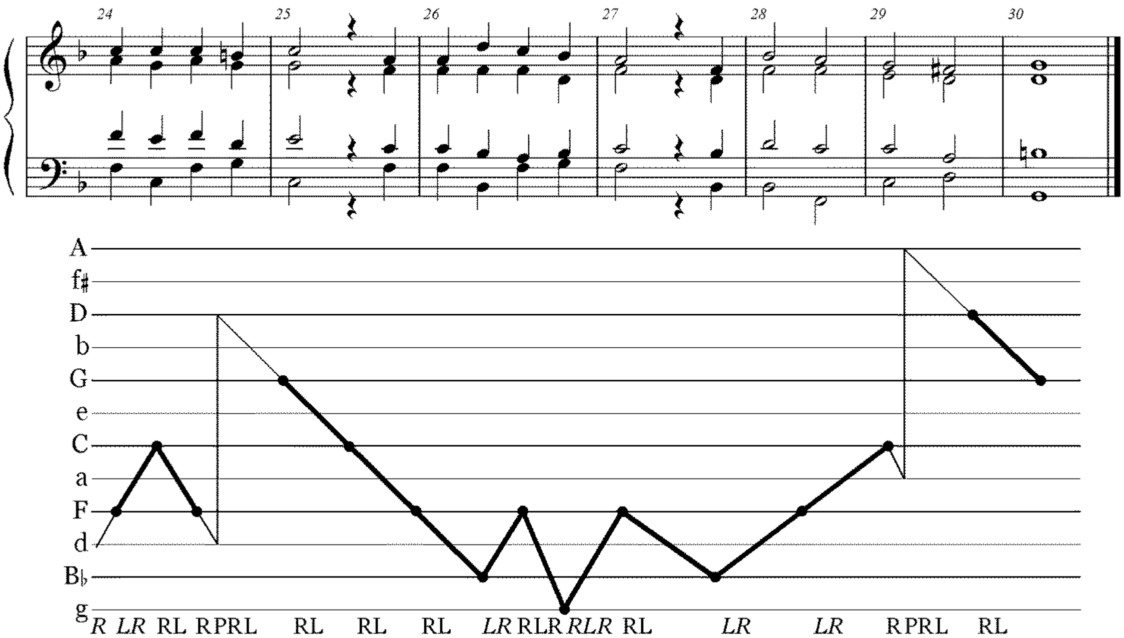
Statistics for *Herr Jesu Christ, mein's Lebens Licht*

2.4. Roland de Lassus, Madrigal *Io ti vorria contar* [1581b]

Only the representation according to the cycle of thirds is proposed for this madrigal, because the description according to the cycle of fifths would not have the simplifying effect as in the examples above. This is also evident in the rather important difference in the statistics concerning the two representations. If the distribution of neo-Riemannian relations (thirds) in the previous examples was similar to that of harmonic vectors (fifths), it was because the harmony there functioned mainly by fifths (two thirds). With Lassus, on the contrary, while the distribution of third relations remains relatively asymmetric (65 % of "normal" relations, against 35 % in the other direction), harmonic vectors are more regularly distributed (55 % against 45 %). The comparison between the two theories appears to indicate a characteristic of this harmony that remains to be elucidated.


1 2 3 4 5 12 13
A
f♯
D
b
G
e
C
a
F
d
B♭
g
E♭
c
RL LR LR RPRL RLP L RPRL LR RL LR LRP PLRLRP


14 15 20 21 22 23
E
c♯
A
f♯
D
b
G
e
C
a
F
d
B♭
g
E♭
c
R RL RPRL RLP PRPRL RL PRLR RL R PRPRL RL RL PRL



Roland de Lassus, Madrigal *Io ti vorria contar*, and analysis
(repetitions of mes. 1-6 and 12-16 are not shown, but are taken account of in the statistics)

Neo-Riemannian transformations

R	31 %	65 %
L	23 %	
P	11 %	
R	16 %	35 %
L	13 %	
P	6 %	

	R	L	P	*R*	*L*	*P*
R		23%	8%	1%		
L	13%				5%	3%
P	10%				1%	
R	7%				7%	3%
L				13%		
P		1%	3%	2%		

Harmonic vectors

+4th	32 %	Dominant: 55 %
+2d	18 %	
–3d	5 %	
–4th	36 %	Subdominant: 45 %
–2d	9 %	
+3d		

	+4th	+2d	–3d	*–4th*	*–2d*	*+3d*
+4th	8%	2%	6%	16%	2%	
+2d	12%			6%	2%	
–3d		6%				
–4th	6%			16%	6%	
–2d	8%			2%		
+3d						

Roland de Lassus, *Io ti vorria contar*, statistics